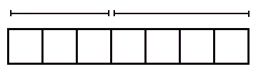
Lesson 7: Algebraic Expressions—The Commutative and Associative Properties

Classwork

Exercise 1

Suzy draws the following picture to represent the sum :

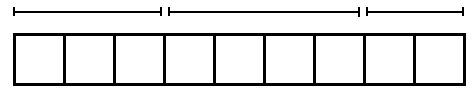


Ben looks at this picture from the opposite side of the table and says, “You drew .”

Explain why Ben might interpret the picture this way.

Exercise 2

Suzy adds more to her picture and says, “The picture now represents .”



How might Ben interpret this picture? Explain your reasoning.

Exercise 3

Suzy then draws another picture of squares to represent the product . Ben moves to the end of the table and says, “From my new seat, your picture looks like the product .”

What picture might Suzy have drawn? Why would Ben see it differently from his viewpoint?

Exercise 4

Draw a picture to represent the quantity that also could represent the quantity when seen from a different viewpoint.

**Four Properties of Arithmetic:**

The Commutative Property of Addition: If and are real numbers, then .

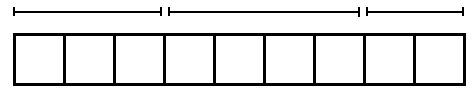
The Associative Property of Addition: If , , and are real numbers, then .

The Commutative Property of Multiplication: If and are real numbers, then .

The Associative Property of Multiplication: If , , and are real numbers, then .

Exercise 5

Viewing the diagram below from two different perspectives illustrates that equals .

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Is it true for all real numbers ,, and that should equal ?

(Note: The direct application of the associative property of addition only gives .)

Exercise 6

Draw a flow diagram and use it to prove that for all real numbers , , and .

Exercise 7

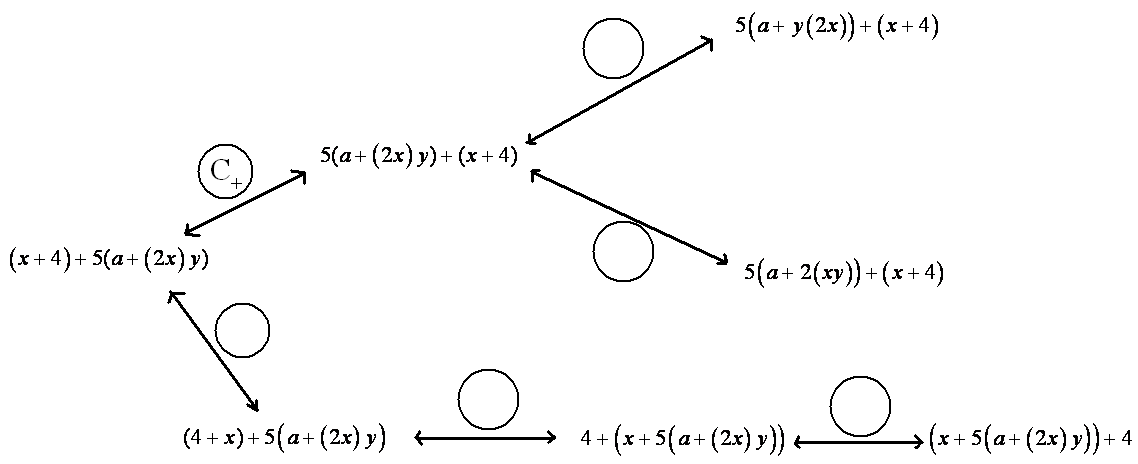
Use these abbreviations for the properties of real numbers and complete the flow diagram.

for the commutative property of addition

for the commutative property of multiplication

for the associative property of addition

for the associative property of multiplication



Exercise 8

Let ,,,and be real numbers. Fill in the missing term of the following diagram to show that is sure to equal .



**Numerical Symbol**: A *numerical symbol* is a symbol that represents a specific number.

For example, ,,,,,,,, are numerical symbols used to represent specific points on the real number line.

**Variable Symbol**: A *variable symbol* is a symbol that is a placeholder for a number.

It is possible that a question may restrict the type of number that a placeholder might permit; e.g., integers only or positive real numbers.

**Algebraic Expression**: An *algebraic expression* is either

1. A numerical symbol or a variable symbol, or
2. The result of placing previously generated algebraic expressions into the two blanks of one of the four operators or into the base blank of an exponentiation with an exponent that is a rational number.

Two algebraic expressions are *equivalent* if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the properties of rational exponents to components of the first expression.

**Numerical Expression**: A *numerical expression* is an algebraic expression that contains only numerical symbols (no variable symbols), which evaluate to a single number.

The expression, , is not a numerical expression.

**Equivalent Numerical Expressions**: Two numerical expressions are *equivalent* if they evaluate to the same number.

Note that and , for example, are equivalent numerical expressions (they are both ) but and are not equivalent expressions.

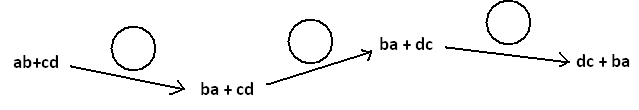
Lesson Summary

The commutative and associative properties represent key beliefs about the arithmetic of real numbers. These properties can be applied to algebraic expressions using variables that represent real numbers.

Two algebraic expressions are ***equivalent*** if we can convert one expression into the other by repeatedly applying the commutative, associative, and distributive properties and the properties of rational exponents to components of the first expression.

Problem Set

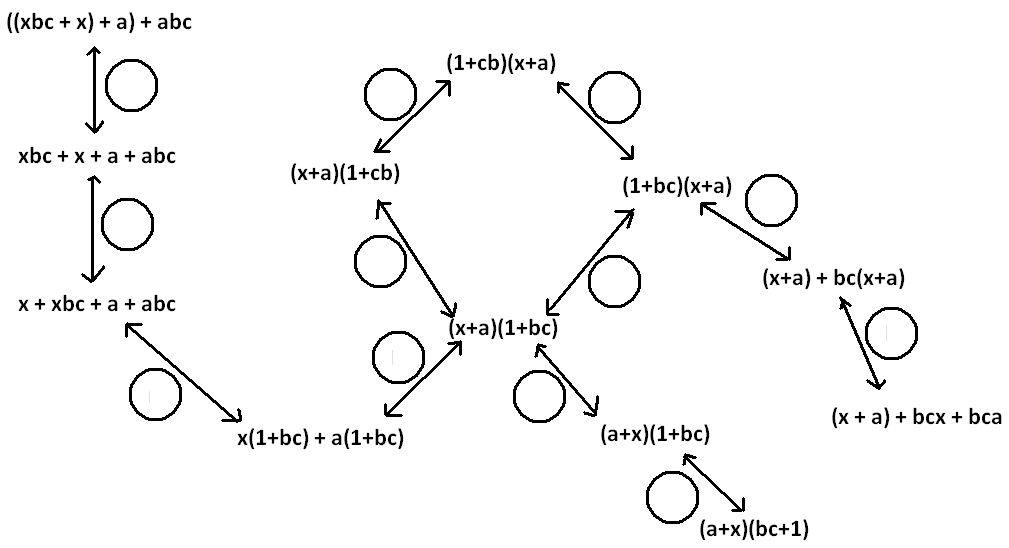
1. The following portion of a flow diagram shows that the expression is equivalent to the expression .



Fill in each circle with the appropriate symbol: Either (for the “commutative property of addition”) or (for the “commutative property of multiplication”).

1. Fill in the blanks of this proof showing that is equivalent . Write either “commutative property,” “associative property,” or “distributive property” in each blank.

1. Fill in each circle of the following flow diagram with one of the letters: C for commutative property (for either addition or multiplication), A for associative property (for either addition or multiplication), or D for distributive property.



1. What is a quick way to see that the value of the sum is ?
   1. If and , what is the value of the product ?
   2. Give some indication as to how you used the commutative and associative properties of multiplication to evaluate in part (a).
   3. Did you use the associative and commutative properties of addition to answer Question 4?
2. The following is a proof of the algebraic equivalency of and . Fill in each of the blanks with either the statement “commutative property” or “associative property.”

1. Write a mathematical proof of the algebraic equivalency of and .
   1. Suppose we are to play the -number game with the symbols , , , and to represent numbers, each used at most once, combined by the operation of addition ONLY. If we acknowledge that parentheses are unneeded, show there are essentially only expressions one can write.
   2. How many answers are there for the multiplication ONLY version of this game?
2. Write a mathematical proof to show that is equivalent to .
3. Recall the following rules of exponents:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |

Here , , , and are real numbers with and non-zero.

Replace each of the following expressions with an equivalent expression in which the variable of the expression appears only once with a positive number for its exponent. (For example, is equivalent to .)

Optional Challenge:

1. Grizelda has invented a new operation that she calls the “average operator.” For any two real numbers and , she declares to be the average of and :
   1. Does the average operator satisfy a commutative property? That is, does for all real numbers and ?
   2. Does the average operator distribute over addition? That is, does for all real numbers , , and ?