



## Lesson 13: Some Potential Dangers when Solving Equations

### Student Outcomes

- Students learn “if-then” moves using the properties of equality to solve equations. Students also explore moves that may result in an equation having more solutions than the original equation.

In previous lessons we have looked at techniques for solving equations, a common theme throughout algebra. In this lesson, we will examine some potential dangers where our intuition about algebra may need to be examined.

### Exercise 1 (4 minutes)

Give students a few minutes to answer the questions individually. Then, elicit responses from students.

#### Exercise 1

- a. Describe the property used to convert the equation from one line to the next:

$$x - 1 - x + 2x - 4 = 8x - 24 - x^2$$

$$x - x^2 + 2x - 4 = 8x - 24 - x^2$$

*distributive property*

$$x + 2x - 4 = 8x - 24$$

*added  $x^2$  to both sides of the equation*

$$3x - 4 = 8x - 24$$

*collected like terms*

$$3x + 20 = 8x$$

*added 24 to both sides of the equation*

$$20 = 5x$$

*subtracted  $3x$  from both sides of the equation*

In each of the steps above, we applied a property of real numbers and/or equations to create a new equation.

- b. Why are we sure that the initial equation  $x - 1 - x + 2x - 4 = 8x - 24 - x^2$  and the final equation  $20 = 5x$  have the same solution set?

*We established last class that making use of the commutative, associative, and distributive properties, and properties of equality to “rewrite” an equation does not change the solution set of the equation.*

- c. What is the common solution set to all these equations?

$$x = 4$$

- Do we know for certain that  $x = 4$  is the solution to every equation shown? Explain why.
  - Have students verify this by testing the solution in a couple of the equations.

**Exercise 2 (4 minutes)**

Work through the exercise as a class. Perhaps have one student writing the problem on the board and one student writing the operation used in each step as the class provides responses.

Emphasize that the solution obtained in the last step is the same as the solution to each of the preceding equations. The moves made in each step did not change the solution set.

Consider:

- Students may point out that they solved the equation in a different way but got the same answer. Consider allowing them to show their approach and discuss whether or not it was algebraically sound.

**Exercise 2**

Solve the equation for  $x$ . For each step, describe the operation used to convert the equation.

$3x$	
$- 8 - 3x - 1$	
$= x + 19$	<i>distributive property</i>
$3x$	<i>commutative property/collected like terms</i>
$- 8 - 3x - 1$	<i>distributive property</i>
$= x + 19$	<i>commutative property/collected like terms</i>
$3x$	<i>subtracted <math>x</math> from both sides</i>
$- 8 - 3x + 3$	<i>added 11 to both sides</i>
$= x + 19$	<i>divided both sides by 5</i>
$3x - 11 - 3x$	
$= x + 19$	
$3x - 11 + 3x$	
$= x + 19$	
$6x - 11$	
$= x + 19$	
$5x - 11 = 19$	
$5x = 30$	
$x = 6$	

**Exercise 3 (8 minutes)**

**Exercise 3**

Solve each equation for  $x$ . For each step, describe the operation used to convert the equation.

- a.  $7x - [4x - 3(x - 1)] = x + 12$   
 {3}
- b.  $2[2(3 - 5x) + 4] = 5[2(3 - 3x) + 2]$   
 {2}

$$\text{c. } \frac{1}{2} 18 - 5x = \frac{1}{3}(6 - 4x)$$

{6}

Note with the class that students may have different approaches that arrived at the same answer.

Ask students how they handled the fraction in part (c).

- Was it easier to use the distributive property first or multiply both sides by 6 first?

### Discussion (10 minutes)

Use the following sample dialog to inspire a similar exchange between you and your students where you play the part of Mike, suggesting ideas of actions you could perform on both sides of an equation that would not predictably preserve the solution set of the original equation. Start by asking students to summarize what they have been studying over the last two lessons and then make Mike's first suggestion. Be sure to provide more than one idea for things that could be done to both sides of an equation that might result in solutions that are not part of the solution set for the original equation, and conclude with an affirmation that you can try anything, but you will have to check to see if your solutions work with the original equation.

- Fergus says, "Basically, what I've heard over the last two lessons is that whatever you do to the left side of the equation, do the same thing to the right side. Then solutions will be good."
- Lulu says, "Well, we've only said that for the properties of equality – adding quantities and multiplying by non-zero quantities. (And associative, commutative, and distributive properties too.) Who knows if it is true in general?"

- Mike says, "Okay ... Here's an equation:

$$\frac{x}{12} = \frac{1}{3}$$

- If I follow the idea, "Whatever you do to the left, do to the right as well," then I am in trouble. What if I decide to remove the denominator on the left and also remove the denominator on the right. I get  $x = 1$ . Is that a solution?"
- Fergus replies, "Well, that is silly. We all know that is a wrong thing to do. You should multiply both sides of that equation by 12. That gives  $x = 4$ , and that does give the correct solution."
- Lulu says, "Okay Fergus, you have just acknowledged that there are some things we can't do! Even if you don't like Mike's example, he's got a point."
- Mike or another student says, "What if I take your equation and choose to square each side. This gives

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$$\frac{x^2}{144} = \frac{1}{9}$$

- Multiplying through by 144 gives  $x^2 = \frac{144}{9} = 16$ , which has solutions  $x = 4$  AND  $x = -4$ .”
- Fergus responds, “Hmmm. Okay I do see the solution  $x = 4$ , but the appearance of  $x = -4$  as well is weird.”
- Mike says, “Lulu is right. Over the past two days we have learned that using the commutative, associative, and distributive properties, along with the properties of equality (adding and multiplying equations throughout) definitely DOES NOT change solution sets. BUT if we do anything different from this we might be in trouble.”
- Lulu continues, “Yeah! Basically when we start doing unusual operations on an equation, we are really saying that **\*\*IF\*\*** we have a solution to an equation, then it should be a solution to the next equation as well. BUT remember, it could be that there was no solution to the first equation anyway!”
- Mike says, “So feel free to start doing weird things to both sides of an equation if you want (though you might want to do sensible weird things!), but all you will be getting are possible CANDIDATES for solutions. You are going to have to check at the end if they really are solutions.”

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### Exercises 4–7 (12 minutes)

Allow students to work through Exercises 4–7 either individually or in pairs. Point out that they are trying to determine what impact certain moves have on the solution set of an equation.

#### Exercise 4

Consider the equations  $x + 1 = 4$  and  $x + 1^2 = 16$ .

- a. Verify that  $x = 3$  is a solution to both equations.

$$\begin{aligned} 3 + 1 &= 4 \\ 3 + 1^2 &= 16 \end{aligned}$$

- b. Find a second solution to the second equation.

$$x = -5$$

- c. Based on your results, what effect does squaring both sides of an equation appear to have on the solution set?

*Answers will vary. The new equation seems to retain the original solution and add a second solution.*

#### Exercise 5

Consider the equations  $x - 2 = 6 - x$  and  $(x - 2)^2 = (6 - x)^2$ .

- a. Did squaring both sides of the equation affect the solution sets?

*No,  $x = 4$  is the only solution to both equations.*

- b. Based on your results, does your answer to part (c) of the previous question need to be modified?

*The new equation retains the original solution and may add a second solution.*

Exercise 6

Consider the equation  $x^3 + 2 = 2x^2 + x$ .

- a. Verify that  $x = 1$ ,  $x = -1$ , and  $x = 2$  are each solutions to this equation.

$$1^3 + 2 = 2 \cdot 1^2 + 1 \text{ True}$$

$$(-1)^3 + 2 = 2(-1)^2 + -1 \text{ True}$$

$$(2)^3 + 2 = 2(2)^2 + 2 \text{ True}$$

- b. Bonzo decides to apply the action “Ignore the exponents” on each side of the equation. He gets  $x + 2 = 2x + x$ . In solving this equation, what does he obtain? What seems to be the problem with his technique?

$x = 1$ ; *The problem is that he only finds one of the three solutions to the equation.*

- c. What would Bonzo obtain if he applied his “method” to the equation  $x^2 + 4x + 2 = x^4$ ? Is it a solution to the original equation?

$x = -\frac{1}{2}$ ; *No, it is not a solution to the original equation.*

Exercise 7

Consider the equation  $x - 3 = 5$ .

- a. Multiply both sides of the equation by a constant, and show that the solution set did not change.

$$7(x - 3) = 7(5)$$

$$7(8 - 3) = 7(5)$$

$$7 \cdot 5 = 7(5)$$

Now, multiply both sides by  $x$ .

$$x \cdot x - 3 = 5x$$

- b. Show that  $x = 8$  is still a solution to the new equation.

$$8(8 - 3) = 5(8)$$

$$8(5) = 5(8)$$

- c. Show that  $x = 0$  is also a solution to the new equation.

$$0 \cdot 0 - 3 = 5(0)$$

Now, multiply both sides by the factor  $x - 1$ .

$$(x - 1)x(x - 3) = 5x(x - 1)$$

- d. Show that  $x = 8$  is still a solution to the new equation.

$$\begin{array}{r} 8 - 1 \quad 8 \quad 8 - 3 = 5 \quad 8 \quad 8 - 1 \\ 7 \quad 8 \quad 5 = 5 \quad 8 \quad 7 \end{array}$$

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- e. Show that  $x = 1$  is also a solution to the new equation.

$$\begin{aligned}(1 - 1)(1)(1 - 3) &= 5(1)(1 - 1) \\ 0(1)(-2) &= 5(1)(0) \\ 0 &= 0\end{aligned}$$

- f. Based on your results, what effect does multiplying both sides of an equation by a constant have on the solution set of the new equation?

*Multiplying by a constant does not change the solution set.*

- g. Based on your results, what effect does multiplying both sides of an equation by a variable factor have on the solution set of the new equation?

*Multiplying by a variable factor could produce additional solution(s) to the solution set.*

Review answers and discuss the following points:

- Does squaring both sides of an equation change the solution set?
  - *Sometimes but not always!*
- For Exercise 6, was it just luck that Bonzo got one out of the three correct answers?
  - *Yes, in part (c), the answer obtained is not a solution to the original equation.*

*Scaffold:*

Have early finishers explore the idea of cubing both sides of an equation. If  $x = 2$ , then  $x^3 = 8$ . If  $x^3 = 8$ , can  $x$  equal any real number besides 2?

Consider having students make up another problem to verify.

- What effect did multiplying both sides by a variable factor have on the solution set?
  - *In our case, it added another solution to the solution set.*
- Can we predict what the second solution will be?

Have students make up another problem to test the prediction.

### Closing (2 minutes)

- What moves have we seen that do not change the solution set of an equation?
- What moves did change the solution set?
- What limitations are there to the principle “whatever you do to one side of the equation, you must do to the other side?”

#### Lesson Summary

Assuming that there is a solution to an equation, applying the distributive, commutative, and associative properties and the properties of equality to equations will not change the solution set.

Feel free to try doing other operations to both sides of an equation, but be aware that the new solution set you get contains possible candidates for solutions. You have to plug each one into the original equation to see if it really is a solution to your original equation.

**Exit Ticket (5 minutes)**

Name \_\_\_\_\_

Date \_\_\_\_\_

## Lesson 13: Some Potential Dangers when Solving Equations

### Exit Ticket

1. Solve the equation for  $x$ . For each step, describe the operation and/or properties used to convert the equation.

$$5(2x - 4) - 11 = 4 + 3x$$

2. Consider the equation  $x + 4 = 3x + 2$ .

- a. Show that adding  $x + 2$  to both sides of the equation does not change the solution set.
- b. Show that multiplying both sides of the equation by  $x + 2$  adds a second solution of  $x = -2$  to the solution set.



## Exit Ticket Sample Responses

1. Solve the equation for  $x$ . For each step, describe the operation and/or properties used to convert the equation.

$$5(2x - 4) - 11 = 4 + 3x$$

*Solution set is {5}.*

2. Consider the equation  $x + 4 = 3x + 2$ .

- a. Show that adding  $x + 2$  to both sides of the equation does not change the solution set.

$$x + 4 = 3x + 2$$

$$4 = 2x + 2$$

$$2 = 2x$$

$$1 = x$$

$$x + 4 + x + 2 = 3x + 2 + x + 2$$

$$2x + 6 = 4x + 4$$

$$2 = 2x$$

$$1 = x$$

- b. Show that multiplying both sides of the equation by  $x + 2$  adds a second solution of  $x = -2$  to the solution set.

$$(x + 2)(x + 4) = (x + 2)(3x + 2)$$

$$(-2 + 2)(-2 + 4) = (-2 + 2)(3(-2) + 2)$$

$$0 \cdot 2 = (0)(-4)$$

$$0 = 0$$

## Problem Set Sample Responses

1. Solve each equation for  $x$ . For each step, describe the operation used to convert the equation. How do you know that the initial equation and the final equation have the same solution set?

*Steps will vary as in the exit ticket and exercises.*

a.  $\frac{1}{5} 10 - 5x - 2 = \frac{1}{10} x + 1$

*Solution set is  $\frac{39}{11}$ .*

b.  $x^5 + x = x^2 + 3x + 1$

*Solution set is  $\frac{1}{2}$ .*

c.  $2x^2 - 2 + 7x = 9x + 2x^3$

*Solution set is 0.*

2. Consider the equation  $x + 1 = 2$ .

*Students should write the new equations and the solution sets:*

- a. Find the solution set.

*Solution set is 1.*

- b. Multiply both sides by  $x + 1$  and find the solution set of the new equation.

*New solution set is  $\pm 1$ .*

- c. Multiply both sides of the original equation by  $x$  and find the solution set of the new equation.

*New solution set is  $0, 1$ .*

3. Solve the equation  $x + 1 = 2x$  for  $x$ . Square both sides of the equation and verify that your solution satisfies this new equation. Show that  $-\frac{1}{3}$  satisfies the new equation but not the original equation.

*The solution of  $x + 1 = 2x$  is  $x = 1$ . The equation obtained by squaring is  $x + 1^2 = 4x^2$ .*

*Let  $x = 1$  in the new equation.  $1 + 1^2 = 4 \cdot 1^2$  is true, so  $x = 1$  is still as solution.*

*Let  $x = -\frac{1}{3}$  in the new equation.  $-\frac{1}{3} + 1^2 = 4 \cdot -\frac{1}{3}^2$  is true, so  $x = -\frac{1}{3}$  is also a solution to the new equation.*

4. Consider the equation  $x^3 = 27$ .

- a. What is the solution set?

*Solution set is  $3$ .*

- b. Does multiplying both sides by  $x$  change the solution set?

*Yes.*

- c. Does multiplying both sides by  $x^2$  change the solution set?

*Yes.*

5. Consider the equation  $x^4 = 16$ .

- a. What is the solution set?

*Solution set is  $-2, 2$ .*

- b. Does multiplying both sides by  $x$  change the solution set?

*Yes.*

- c. Does multiplying both sides by  $x^2$  change the solution set?

*Yes.*