Lesson 15: Solution Sets of Two or More Equations (or Inequalities) Joined by "And" or "Or"

Student Outcomes

- Students describe the solution set of two equations (or inequalities) joined by either "and" or "or" and graph the solution set on the number line.

Classwork

Exercise 1 (6 minutes)

It may be helpful to some students to review some of the vocabulary used here, such as compound sentence (a sentence that contains at least two clauses) or declarative sentence (a sentence in the form of a statement).

Give students a few minutes to work on the exploration independently and then 1 minute to compare answers with a partner. Discuss results as a class, particularly the difference between separating the declarations by "and" and by "or."

Exercise 1

Determine whether each claim given below is true or false.

a. **Right now, I am in math class and English class.**

False

b. **Right now, I am in math class or English class.**

True (assuming they are answering this in class)

c. $3 + 5 = 8$ **and** $5 < 7 - 1$.

True

d. $10 + 2 \neq 12$ **and** $8 - 3 > 0$

False

e. $3 < 5 + 4$ **or** $6 + 4 = 9$.

True

f. $16 - 20 > 1$ **or** $5.5 + 4.5 = 11$

False

These are all examples of declarative compound sentences.

g. **When the two declarations in the sentences above were separated by "and," what had to be true to make the statement true?**

Both declarations had to be true.

h. **When the two declarations in the sentences above were separated by "or," what had to be true to make the statement true?**

At least one declaration had to be true.

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Discuss the following points with students:

- The word "and" means the same thing in a compound mathematical sentence as it does in an English sentence.
- If two clauses are separated by "and," both clauses must be true for the entire compound statement to be deemed true.
- The word "or" also means a similar thing in a compound mathematical sentence as it does in an English sentence. However, there is an important distinction: In English the word "or" is commonly interpreted as the exclusive or, one condition or the other is true, but not both. In mathematics, either or both could be true.
- If two clauses are separated by "or," one or both of the clauses must be true for the entire compound statement to be deemed true.

Example 1 (5 minutes)

Work through the four examples as a class.

Example 1

Solve each system of equations and inequalities.

a. $x + 8 = 3$ or $x - 6 = 2$

$x = -5$ *or* $x = 8$

$\{-5, 8\}$

b. $4x - 9 = 0$ or $3x + 5 = 2$

$x = \frac{9}{4}$ *or* $x = -1$

$-1, \frac{9}{4}$

c. $x - 6 = 1$ and $x + 2 = 9$

$x = 7$ *and* $x = 7$

$\{7\}$

d. $2w - 8 = 10$ and $w > 9$.

The empty set.

$\emptyset$

Exploratory Challenge/Exercise 2 (10 minutes)

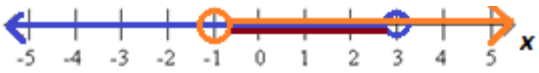
Provide students with colored pencils and allow them a couple of minutes to complete (a) through (c). Then, stop and discuss the results.

Exercise 2

a. **Using a colored pencil, graph the inequality $x < 3$ on the number line below.**

b. **Using a different colored pencil, graph the inequality $x > -1$ on the number line below.**

c. **Using a third colored pencil, darken the section of the number line where $x < 3$ and $x > -1$.**



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- In order for the compound sentence $x > -1$ and $x < 3$ to be true, what has to be true about x?
  - *x has to be both greater than -1 and less than 3. (Students might also verbalize that it must be between -1 and 3, not including the points -1 and 3.)*
- On the graph, where do the solutions lie?
  - *Between -1 and 3, not including the points -1 and 3*

Have students list some of the solutions to the compound inequality. Make sure to include examples of integer and non-integer solutions.

- How many solutions are there to this compound inequality?
  - *An infinite number*

Introduce the abbreviated way of writing this sentence:

- Sometimes this is written as $-1 < x < 3$.

Use this notation to further illustrate the idea of x representing all numbers ***strictly between*** -1 and 3.

Allow students a couple of minutes to complete (d) *through* (f). Then, stop and discuss the results.

d. Using a colored pencil, graph the inequality $x < -4$ on the number line below.

e. Using a different colored pencil, graph the inequality $x > 0$ on the number line below.

f. Using a third colored pencil, darken the section of the number line where $x < -4$ or $x > 0$.

- In order for the compound sentence $x < -4$ or $x > 0$ to be true, what has to be true about x?
  - *It could either be less than -4, or it could be greater than 0.*
- On the graph, where do the solutions lie?
  - *To the left of -4 and to the right of 0*

Have students list solutions to the compound inequality. Make sure to include examples of integer and non-integer solutions.

- How many solutions are there to this compound inequality?
  - *Infinitely many*
- Would it be acceptable to abbreviate this compound sentence as follows: $0 < x < -4$?
  - *No.*
- Explain why not.
  - *Those symbols suggest that x must be greater than zero and less than -4 at the same time, but the solution is calling for x to be either less than -4 or greater than zero.*

Allow students a couple of minutes to complete (g) *through* (i) and discuss answers.

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g. Graph the compound sentence $x > -2$ or $x = -2$ on the number line below.

h. How could we abbreviate the sentence $x > -2$ or $x = -2$?

$x \geq -2$

i. Rewrite $x \leq 4$ as a compound sentence and graph the solutions to the sentence on the number line below.

$x < 4$ *or* $x = 4$

Example 2 (3 minutes)

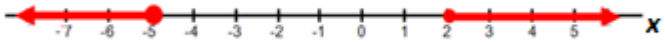
Work through Example 2 as a class.

Example 2

Graph each compound sentence on a number line.

a. $x = 2$ or $x > 6$

b. $x \leq -5$ or $x \geq 2$



Rewrite as a compound sentence and graph the sentence on a number line.

c. $1 \leq x \leq 3$

$x \geq 1$ *and* $x \leq 3$

Exercise 3 (5 minutes)

Give students a couple of minutes to read through Exercise 3 and try it independently before comparing answers with a neighbor or discussing as a class.

Exercise 3

Consider the following two scenarios. For each, specify the variable and say, "W is the width of the rectangle," for example, and write a compound inequality that represents the scenario given. Draw its solution set on a number line.

Scenario	Variable	Inequality	Graph
a. Students are to present a persuasive speech in English class. The guidelines state that the speech must be at least 7 minutes but not exceed 12 minutes.	*Let x = length of time of the speech.*	$x \geq 7$ *and* $x \leq 12$ $7 \leq x \leq 12$	

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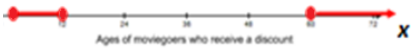
b. Children and senior citizens receive a discount on tickets at the movie theater. To receive a discount, a person must be between the ages of 2 and 12, including 2 and 12, or 60 years of age or older.	*Let x = age of moviegoer who receives a discount.*	$2 \leq x \leq 12$ *or* $x \geq 60$	

Scaffolding:

- Ask students to think of other scenarios that could be modeled using a compound inequality.

Exercise 4 (10 minutes)

Give students time to work on the problems, and then allow for sharing of answers, possibly with a neighbor or with the class.

Exercise 4

Determine if each sentence is true or false. Explain your reasoning.

a. $8 + 6 \leq 14$ and $\frac{1}{3} < \frac{1}{2}$.

True

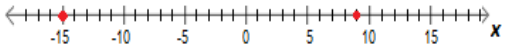
b. $5 - 8 < 0$ or $10 + 13 \neq 23$

True

Solve each system and graph the solution on a number line.

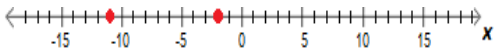
c. $x - 9 = 0$ or $x + 15 = 0$

$\{9, -15\}$



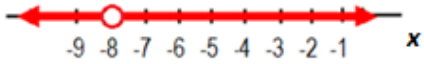
d. $5x - 8 = -23$ or $x + 1 = -10$

$\{-3, -11\}$

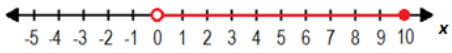


Graph the solution set to each compound inequality on a number line.

e. $x < -8$ or $x > -8$

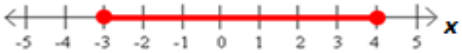


f. $0 < x \leq 10$



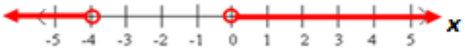
Write a compound inequality for each graph.

g.



$-3 \leq x \leq 4$

h.



$x < -4$ *or* $x > 0$

i. **A poll shows that a candidate is projected to receive 57% of the votes. If the margin for error is plus or minus 3%, write a compound inequality for the percentage of votes the candidate can expect to get.**

let x = percentage of votes $54 \leq x \leq 60$

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j. **Mercury is one of only two elements that is liquid at room temperature. Mercury is non-liquid for temperatures less than $-38.0°$F or greater than $673.8°$F. Write a compound inequality for the temperatures at which mercury is non-liquid.**

Let x = temperatures (in degrees F) for which mercury is non-liquid $x < -38$ *or* $x > 673.8$

As an extension, students can come up with ways to alter parts (a) and (b) to make them false compound statements. Share several responses.

Ask the following:

- What would be a more concise way of writing the sentence for part (e)?
  - $x \neq 8$
- For part (f), list some numbers that are solutions to the inequality.
- What is the largest possible value of x?
  - 10

MP.2

- What is the smallest possible value of x?
  - This is tougher to answer. x can be infinitely close to 0 but cannot equal zero. Therefore, there is no absolute smallest value for x in this case.

For parts (i) and (j), make sure students specify what the variable they choose represents.

Scaffolding:

- The other element that is liquid at room temperature is bromine. Students could be asked to look up the temperatures at which bromine is non-liquid and write a similar compound inequality.

Closing (2 minutes)

Lead a conversation on the idea that in math, as in English, it is important that we are precise in our use of language and that we are able to read (and comprehend) and write mathematical sentences. Ask students to give examples to justify why the precision is important in math, and why it is important in English.

MP.6

Reinforce that, in mathematical sentences, like in English sentences, a compound sentence separated by

AND is true if <u>both clauses are true</u>.

OR is true if <u>at least one of the clauses is true</u>.

Lesson Summary

In mathematical sentences, like in English sentences, a compound sentence separated by

AND is true if _____.

OR is true if _____.

Exit Ticket (5 minutes)

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Name _____ Date_____

Lesson 15: Solution Sets of Two or More Equations (or Inequalities) Joined by "And" or "Or"

Exit Ticket

1.

a. Solve the system and graph the solution set on a number line.

$x - 15 = 5$ or $2x + 5 = 1$

b. Write a different system of equations that would have the same solution set.

2. Swimming pools must have a certain amount of chlorine content. The United States standard for safe levels of chlorine in swimming pools is at least 1 part per million and no greater than 3 parts per million. Write a compound inequality for the acceptable range of chlorine levels.

3. Consider each of the following compound sentences:

$x < 1$ and $x > -1$ $x < 1$ or $x > -1$

Does the change of word from "and" to "or" change the solution set?

Use number-line graphs to support your answer.

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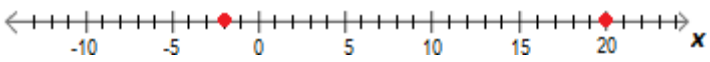
Exit Ticket Sample Solutions

1.

a. Solve the system and graph the solution set on a number line.

$x - 15 = 5$ or $2x + 5 = 1$

$x = 20$ or $x = -2$ $\{-2, 20\}$



b. Write a different system of equations that would have the same solution set.

sample answer: $x - 20 = 0$ or $x + 2 = 0$

2. Swimming pools must have a certain amount of chlorine content. The United States standard for safe levels of chlorine in swimming pools is at least 1 part per million and no greater than 3 parts per million. Write a compound inequality for the acceptable range of chlorine levels.

Let x = chlorine level in a swimming pool (in parts per million) $1 \leq x \leq 3$

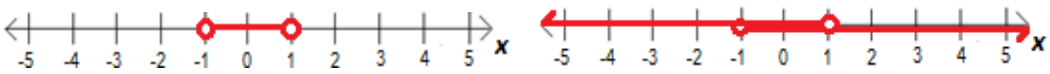
3. Consider each of the following compound sentences:

$x < 1$ and $x > -1$ $x < 1$ or $x > -1$

Does the change of word from "and" to "or" change the solution set?

Use number-line graphs to support your answer.

For the first sentence, both statements must be true, so x can only equal values that are both greater than -1 and less than 1. For the second sentence, only one statement must be true, so x must be greater than -1 or less than 1. This means x can equal any number on the number line.



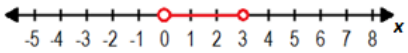
Problem Set Sample Solutions

1. Consider the inequality $0 < x < 3$.

a. Rewrite the inequality as a compound sentence.

$x > 0$ and $x < 3$

b. Graph the inequality on a number line.



c. How many solutions are there to the inequality? Explain.

There are an infinite number of solutions. x can be any value between 0 and 3, which includes the integer values of 1 and 2 as well as non-integer values. The set of numbers between 0 and 3 is infinite.

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d. What are the largest and smallest possible values for x? Explain.

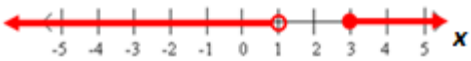
There is no absolute largest or absolute smallest value for x. x can be infinitely close to 0 or to 3 but cannot equal either value.

e. If the inequality is changed to $0 \leq x \leq 3$, then what are the largest and smallest possible values for x?

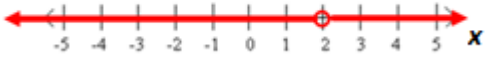
In this case, we can define the absolute maximum value to be 3 and the absolute minimum value to be 0.

Write a compound inequality for each graph.

2. $x < 1$ *or* $x \geq 3$



3. $x < 2$ *or* $x > 2$, which can be written as $x \neq 2$



Write a single or compound inequality for each scenario.

4. The scores on the last test ranged from 65% to 100%.

x = scores on last test $65 \leq x \leq 100$

5. To ride the roller coaster, one must be at least 4 feet tall.

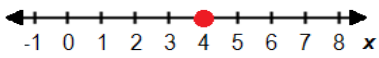
x = height (in feet) to ride the roller coaster $x \geq 4$

6. Unsafe body temperatures are those lower than 96°F or above 104°F.

x = body temperature (in degrees F) that are unsafe $x < 96$ *or* $x > 104$

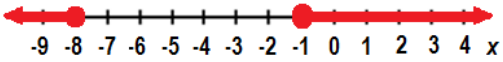
Graph the solution(s) to each of the following on a number line.

7. $x - 4 = 0$ and $3x + 6 = 18$

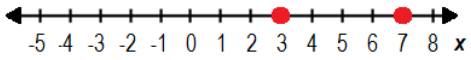


8. $x < 5$ and $x \neq 0$

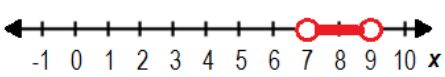
9. $x \leq -8$ or $x \geq -1$



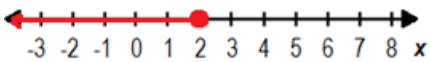
10. $3(x - 6) = 3$ or $5 - x = 2$



11. $x < 9$ and $x > 7$



12. $x + 5 < 7$ or $x = 2$



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