Equation of a Circle

<u>Objectives:</u>

-Write the equation of a circle.

-Graph a circle on the coordinate plane.

<u>Common Core Standards:</u>

G.GPE.1 - Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.6 - Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

A.CED.4 - Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Equation of a Circle

-All points are equidistant from the center (Definition of a Circle)

-You can use the Pythagorean Theorem/Distance Formula to derive the equation of a circle

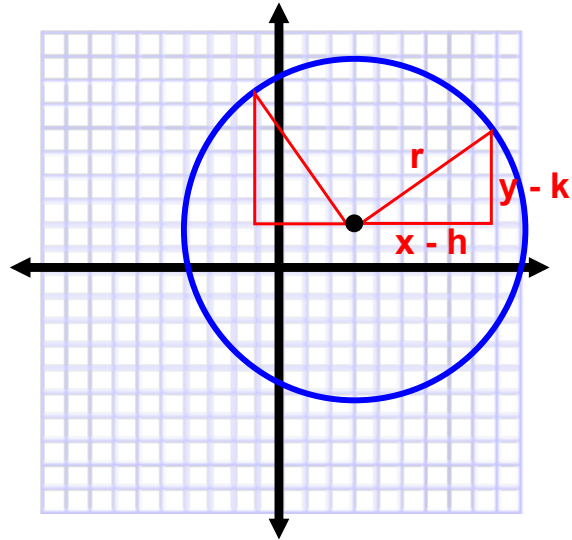   -The radius is always the same, no matter where the triangle is drawn.
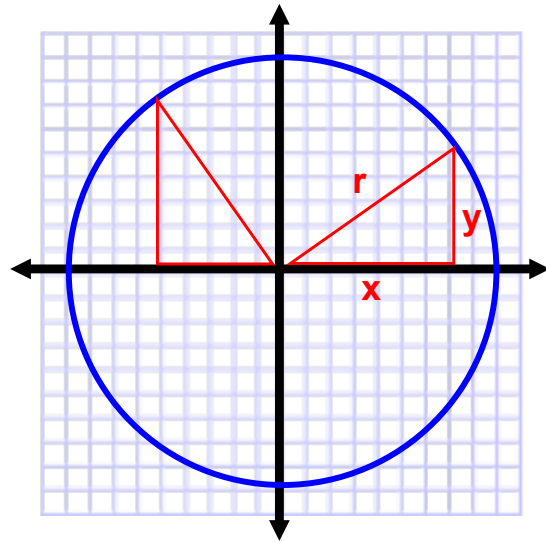
$x^2 + y^2 = r^2$

Example: $x^2 + y^2 = 8^2$

$x^2 + y^2 = 64$

Center (0, 0) and Radius of 8

What if the center of the circle is not at (0,0)?

-Let the center be represented by the point (h, k)

$(x - h)^2 + (y - k)^2 = r^2$

Example: $(x - 3)^2 + (y - 2)^2 = 6^2$

$(x - 3)^2 + (y - 2)^2 = 36$

Center (3, 2) and Radius of 6

Equation of a Circle:

$(x - h)^2 + (y - k)^2 = r^2$

Center (h, k)

Radius = r

Suggested Explorations for Deeper Understanding:

Exploring the Equation of a Circle Walkthrough (By: Jennifer Wilson)
http://easingthehurrysyndrome.wordpress.com/2013/06/18/exploring-the-equation-of-a-circle/

Exploring the Equation of a Circle (By: Texas Instruments)
http://education.ti.com/en/timathnspired/us/detail?id=C52E12D134494960A47F7ACA43BB176B&t=6BE0FB24FE4348FD912CB3ADC4A038FB

Find the center and radius of each circle.

$(x - 3)^2 + (y - 7)^2 = 25$

center = (3, 7)

radius = 5

$(x + 5)^2 + (y - 2)^2 = 16$

center = (-5, 2)

radius = 4

$(x - 1)^2 + y^2 = 21$

center = (1, 0)

radius = $\sqrt{21}$

$x^2 + y^2 = 40$

center = (0, 0)

radius = $2\sqrt{10}$

Equation of a Circle:

$(x - h)^2 + (y - k)^2 = r^2$

Center (h, k)

Radius = r

Write the equation of each circle using the information given.

Center at (4, 2) with radius = 7

$(x - 4)^2 + (y - 2)^2 = 49$

Center at (-4, -9) with radius = $\sqrt{10}$

$(x + 4)^2 + (y + 9)^2 = 10$

Center at (0, 8) with radius = $3\sqrt{3}$

$x^2 + (y - 8)^2 = 27$

Center at $(\frac{1}{2}, -2)$ with radius = $2\sqrt{5}$

$(x - \frac{1}{2})^2 + (y + 2)^2 = 20$

Equation of a Circle:

$(x - h)^2 + (y - k)^2 = r^2$

Center (h, k)

Radius = r

Find the center and radius of the circle given by the equation below.

-You will have to complete the square. (From Algebra)

$x^2 + y^2 - 8x + 2y = -8$

$x^2 - 8x +$ _____ $+ y^2 + 2y +$ _____ $= -8$ Reorder.

$x^2 - 8x +$ _16_ $+ y^2 + 2y +$ _1_ $= -8 + 16 + 1$ Complete the square.

$(x - 4)^2 + (y + 1)^2 = 9$ Factor.

Center = (4, -1)

Radius = 3

Completing the Square:

To find the missing value, take half the middle coefficient and square it.

Example: *$x^2 - 8x +$ _____*

$-8 \div 2 = -4$

$(-4)^2 = 16$

Find the center and radius of the circle given by the equation below.

-You will have to complete the square. (From Algebra)

$x^2 + y^2 + 8x - 14y + 40 = 0$

$x^2 + 8x + \underline{\hspace{1cm}} + y^2 - 14y + \underline{\hspace{1cm}} = -40$ Reorder.

$x^2 + 8x + \underline{16} + y^2 - 14y + \underline{49} = -40 + 16 + 49$ Complete the square.

$(x + 4)^2 + (y - 7)^2 = 25$ Factor.

Center = (-4, 7)

Radius = 5

Finding the Radius

-You may have to find the radius by using the distance formula.

Distance Formula:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Write the equation of a circle with a center at (5, -1). A point on the circle is (1, 2).

Find Radius: $r = \sqrt{(1 - 5)^2 + (2 + 1)^2}$
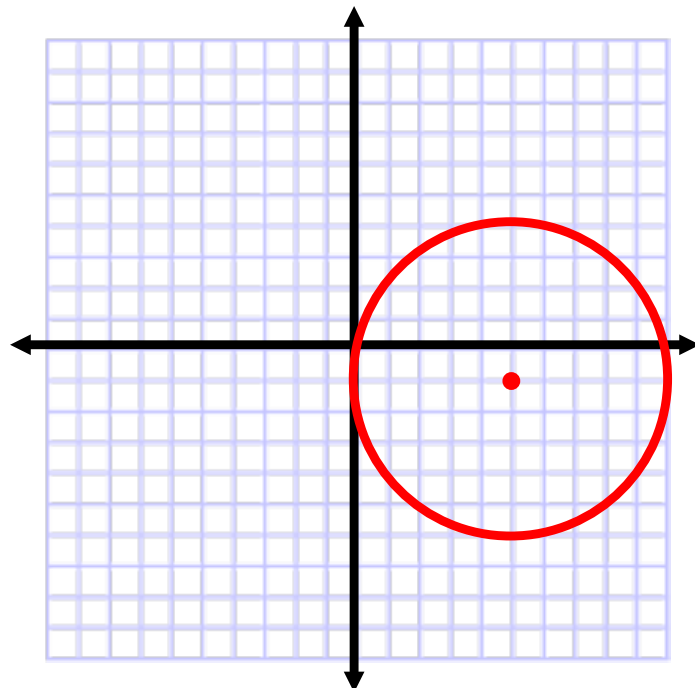
$r = \sqrt{16 + 9}$

$r = \sqrt{25}$

$r = 5$

Equation:

$(x - 5)^2 + (y + 1)^2 = 25$

Write the equation of a circle with a center at (4, -3). A point on the circle is (2, 1).

Find Radius: $r = \sqrt{(2 - 4)^2 + (1 + 3)^2}$

$r = \sqrt{4 + 16}$
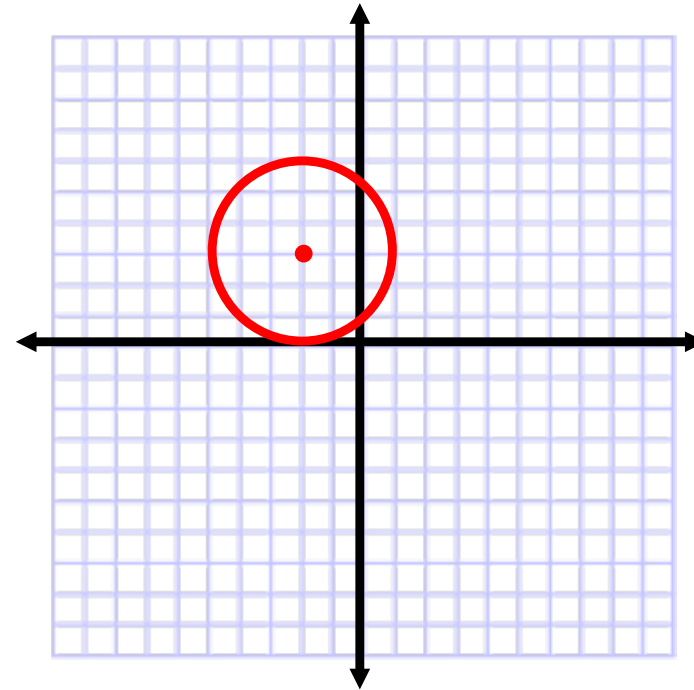
$r = \sqrt{20}$

$r = 2\sqrt{5}$

Equation:

$(x - 4)^2 + (y + 3)^2 = 20$

Find the center and radius of the circle, then graph the equation.

$(x + 2)^2 + (y - 3)^2 = 9$

Center at (-2, 3)

Radius = 3

$(x - 5)^2 + (y + 1)^2 = 25$

Center at (5, -1)

Radius = 5

Find the center and radius of the circle, then graph the equation.
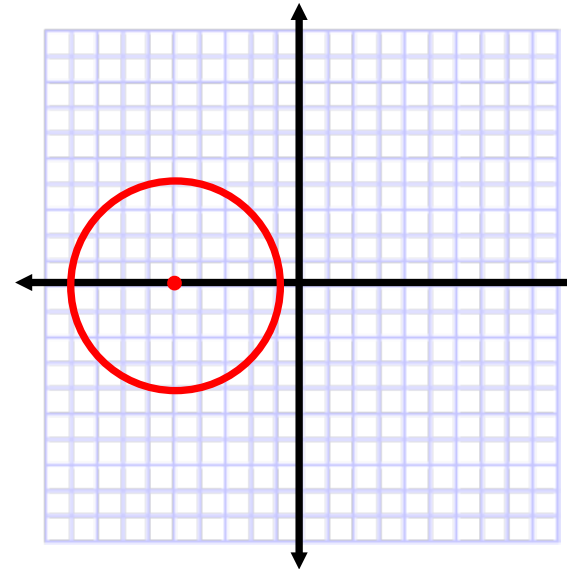
$x^2 + y^2 + 10x + 9 = 0$

$x^2 + 10x +$ _____ $+ y^2 = -9$

$x^2 + 10x + \underline{\ 25\ } + y^2 = -9 + 25$

$(x + 5)^2 + y^2 = 16$

Center at (-5, 0)

Radius = 4

Check: Graphing Calculator
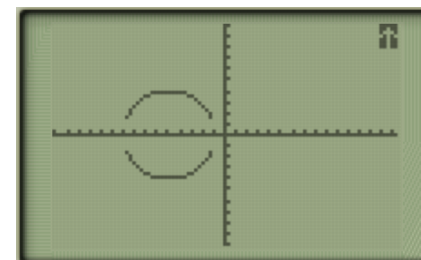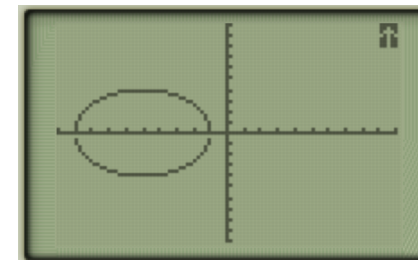
-Put equation in y = form

-Graph both equations

$(x + 5)^2 + y^2 = 16$

$y^2 = 16 - (x + 5)^2$

$y = \pm\sqrt{16 - (x + 5)^2}$

Plot1 Plot2 Plot3
\Y₁◼√(16−(X+5)²)
\Y₂◼ −√(16−(X+5)²)
)
\Y₃=
\Y₄=
\Y₅=

Is a circle a function?

No

Window Adjusted:

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